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*Published in:*  
ArXiv

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*Document Version*  
Publisher's PDF, also known as Version of record

*Publication date:*  
2006

[Link to publication in University of Groningen/UMCG research database](#)

*Citation for published version (APA):*

Gorelik, M. L., Safonov, I. V., & Urin, M. H. (2006). Decay Properties Of The Dipole Isobaric Analog Resonances. *ArXiv*, 69(3).

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# DECAY PROPERTIES OF THE DIPOLE ISOBARIC ANALOG RESONANCES

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A continuum-RPA-based approach is applied to describe the decay properties of isolated dipole isobaric analog resonances in nuclei having not-too-large neutron excess. Calculated for a few resonances in  $^{90}\text{Zr}$  the elastic E1-radiative width and partial proton widths for decay into one-hole states of  $^{89}\text{Y}$  are compared with available experimental data.

PACS numbers: 24.30.Cz, 21.60.Jz, 23.50.+z

## I. INTRODUCTION

In nuclei having not-too-large neutron excess  $N - Z = 2T$  ( $T$  is the ground-state isospin value) the familiar giant dipole resonance (GDR) is splitted into two components, characterized by the isospin values  $T_< = T$  and  $T_> = T + 1$ . The GDR  $T_>$ -component (GDR $_>$ ) is the isobaric analog of the charge-exchange (in the  $\beta^+$ -channel) giant dipole resonance (i.e. of the GDR $^{(+)}$ ). Thus, the GDR $_>$  presents the specific double giant resonance (see, e.g. Ref. [1]). Because of Pauli blocking, the GDR $^{(+)}$  is not strongly collectivized and contains along with the main peak a few low-energy  $1^-$ -states of the neutron-(proton-hole) type [2, 3]. These states is a base for the GDR $_>$  low-energy part, which consists of a few isolated dipole isobaric analog resonances (DIAR) [3]. One of these resonances in  $^{90}\text{Zr}$  ( $E_x = 16.28$  MeV) has been identified in reactions with protons, electrons and photons (see Ref. [4] and references therein). The comprehensive study of the mentioned DIAR with the  $(e, e'p)$ -reaction has been reported in Ref. [4], where along with the form factor for two values of the transferred momentum the elastic E1-radiative width and partial proton widths for decays into one-hole states of  $^{89}\text{Y}$  have been deduced for this resonance. Being motivated by these experimental results, which are still not explained, in the present work we attempt to describe the gross and (direct) decay properties of isolated DIAR within a continuum-RPA(CRPA)-based approach. The basic elements of such description are presented in Sect. II. The applications to the DIAR in  $^{90}\text{Zr}$  are given in Sect. III, where the calculation results are compared with available experimental data.

## II. CONTINUUM-RPA-BASED DESCRIPTION OF DIAR.

### A. DIAR properties

The dipole isobaric analog resonances in a target nucleus  $(Z, N)$  correspond to (2particle-2hole)-type excitations with a small admixture of (proton-(proton-hole))-type states. Namely due to this admixture, E1-decay of the DIAR into the target-nucleus ground state and also (direct) proton decays into one-hole states of the product nucleus  $(Z - 1, N)$  are possible. The description of the DIAR properties is simplified, provided that the isobaric analog state (IAS) is considered as the “ideal” one. The “ideal” IAS exhausts 100% of the Fermi strength and its wave function is generated by the Fermi operator  $\mathcal{T}^{(-)} = \sum_a \tau_a^{(-)}$ . (The real IAR exhausts about 95% of the Fermi strength). In this approximation the wave function  $|s_>, 1M\rangle$  of a GDR $_>$ -component can be expressed via the parent GDR $^{(+)}$ -component wave function  $|s^{(+)}, 1M\rangle$ :

$$|s_>, 1M\rangle = (2T_>)^{-1/2} \mathcal{T}^{(-)} |s^{(+)}, 1M\rangle. \quad (1)$$

In the mean-field approximation, the “ideal” IAS excitation energy equals the Coulomb displacement energy:  $\Delta_C = (2T_>)^{-1} \int U_C(r) n^{(-)}(r) d^3r$ , where  $U_C(r)$  is the mean Coulomb field,  $n^{(-)}(r)$  is the neutron-excess density of the parent nucleus  $(Z - 1, N + 1)$ . (We assume that the IAS energy is independent of the parent state).

Using Eq. (1), we express the GDR $_>$  strength function (corresponding to the single-particle external field  $V^{(0)}(x) = (-1/2)\tau^{(3)}rY_{1M}$ ) via the GDR $^{(+)}$  strength function (corresponding to the external field  $V^{(+)}(x) = \tau^{(+)}rY_{1M}$ ):

$$S_>^{(0)}(\omega) = (2T_>)^{-1} S^{(+)}(\omega_C^{(+)}), \quad (2)$$

where  $\omega$  is the target-nucleus excitation energy,  $\omega_C^{(+)} = \omega - \Delta_C$  [3]. The parent-nucleus dipole strength function  $S^{(+)}(\omega^{(+)})$  can be calculated within any CRPA-based approach (see e.g. Refs. [2, 3]). The cross section of photoabsorption, accompanied by GDR $_>$  excitation, is proportional to the strength function  $S_>^{(0)}(\omega)$ :  $\sigma_a^{>}(\omega) = B\omega S_>^{(0)}(\omega)$ ,

where  $B = (16\pi^3/3)(e^2/\hbar c)$ . This cross section, integrated over an isolated DIAR (with the energy  $\omega_s$ ),  $(\sigma_a^>)_s^{int}$ , determines the E1-radiative width for DIAR decay into the target-nucleus ground state:  $\Gamma_{\gamma 0,s} = (1/3\pi^2)(\omega_s/\hbar c)^2(\sigma_a^>)_s^{int}$ .

Being applied to an isolated DIAR, Eq. (2) allows one to express the (one-dimensional) particle-hole DIAR transition density via the corresponding parent-state transition density:  $\rho_{>,s}(r) = (2T_{>})^{(-1/2)}\rho_s^{(+)}(r)$ . In particular, this transition density can be used for evaluation of the DIAR form factor, which determines the  $(ee')$ -reaction cross section [4]. Within the improved plane wave approximation the form factor equals:

$$|F_s(q)|^2 = 12\pi Z^{-2}\mathcal{Y}^2(Z) \left| \int \rho_{>,s}(r) j_1(qr) dr \right|^2, \quad (3)$$

where  $\mathcal{Y}(Z) = \frac{2\pi Z}{137} \left( 1 - \exp\left(-\frac{2\pi Z}{137}\right) \right)^{-1}$  is the correcting factor (see, e.g., Ref. [7]), and  $j_1(qr)$  is the dipole spherical Bessel function.

The basic quantity in CRPA-based description of DIAR proton decay is the amplitude of the partial  $(\gamma p)$ -reaction, accompanied by DIAR excitation. The poles in the energy dependence of this amplitude correspond to DIAR, while the pole residue is proportional to the product of the amplitudes of the elastic radiative and partial proton widths. Thus, the joint analysis of the corresponding calculated photo-absorption and  $(\gamma p)$ -reaction cross sections allows one to evaluate the partial proton widths for a given DIAR.

## B. CRPA equations

The CRPA expressions for the strength functions  $S^{(\pm)}(\omega^{(\pm)})$  and  $(\gamma p)$ -reaction amplitudes can be obtained taking the CRPA equations in the form consistent with a phenomenological version of the Migdal's finite Fermi-system theory. As applied to description of charge-exchange giant resonances, these equations are given in detail in Refs. [5, 6] and for reader's convenience we use, in the main, the notations from these references.

Of particular importance is the equation for the effective single-particle external field  $\tilde{V}^{(\pm)}(x, \omega^{(\pm)})$ . The difference between the effective field and the corresponding external field  $V^{(\pm)}(x) = V(r)Y_{LM}(\vec{n})\tau^{(\pm)}$  is due to the polarization effect from the particle-hole interaction taken for description of the non-spin-flip excitations in the form:  $F_{p-h} \rightarrow 2F'(\tau^{(+)}\tau^{(-)'} + \tau^{(-)}\tau^{(+)'})\delta(\vec{r} - \vec{r}')$ . The effective-field radial part  $\tilde{V}^{(\pm)}(r, \omega^{(\pm)})$  determines all the main characteristics of a given charge-exchange giant resonance: the strength function  $S_V^{(\pm)}(\omega^{(\pm)})$ , the energy-dependent transition density  $r^{-2}\rho^{(\pm)}(r, \omega^{(\pm)})Y_{LM}$ , and the nucleon-escape amplitude  $M_{V,c}^{(\pm)}(\omega^{(\pm)})$  ( $c$  is the set of the nucleon-decay-channel quantum numbers). In particular, the dipole strength function  $S^{(+)}(\omega^{(+)})$  of Eq. (2) can be directly calculated via the radial part of the corresponding effective field (see, e.g., Eq. (1) of Ref. [5]). Here we show the useful relationships, which have not been exploited in Refs. [2, 3, 5, 6]. The pole representations of the effective-field radial part and strength function are valid in the energy region of each collective neutron-(proton-hole)-type state  $|s^{(+)}, 1M\rangle$  with the excitation energy  $\omega_s^{(+)}$  below the neutron separation threshold:

$$\tilde{V}^{(+)}(r, \omega^{(+)}) \rightarrow \frac{2F'}{r^2} \frac{d_s^{(+)}\rho_s^{(+)}(r)}{\omega^{(+)} - \omega_s^{(+)} + i0}; \quad S^{(+)}(\omega^{(+)}) \rightarrow -\frac{1}{\pi} \text{Im} \frac{(d_s^{(+)})^2}{\omega^{(+)} - \omega_s^{(+)} + i0}. \quad (4)$$

Here,  $\rho_s^{(+)}(r)$  is the transition density of the (stationary) collective state  $|s^{(+)}, 1M\rangle$ , and  $d_s^{(+)} = \int r\rho_s^{(+)}(r)dr$  is the dipole matrix element. In particular, this matrix element determines the partial radiative width  $\Gamma_{\gamma 0,s}$  of the corresponding DIAR (Subsect. II A).

The transition-density elements, which define the transition density  $\rho_s^{(+)}(r) = \sum_{\nu\pi} \rho_{\nu\pi}^{(+s)} \chi_\pi(r) \chi_\nu(r)$ , can be found from the (formally, infinite range) system of the homogeneous equations, which is usually used within a discrete-RPA:

$$\rho_{\nu\pi}^{(+s)} = t_{(\nu)(\pi)}^{(1)} \frac{n_\nu - n_\pi}{\varepsilon_\nu - \varepsilon_\pi - \omega_s^{(+)}} 2F' \int \rho_s^{(+)}(r) \chi_\pi(r) \chi_\nu(r) \frac{dr}{r^2}. \quad (5)$$

Here,  $n_\nu$  ( $n_\pi$ ) are the occupation numbers for neutrons (protons);  $r^{-1}\chi_\nu$  ( $r^{-1}\chi_\pi$ ) are the radial bound-state wave functions for neutrons (protons);  $\nu = n_{r,\nu}, j_\nu, l_\nu$  is the set single-particle quantum numbers;  $\sqrt{3}t_{\nu\pi}^{(1)} = \langle(\nu)||Y_1||(\pi)\rangle$  is the reduced matrix element;  $(\nu) = j_\nu, l_\nu$ ;  $(\pi) = j_\pi, l_\pi$ . Note, that the ratio  $-\rho_{\nu\pi}^{(+s)}/(n_\nu - n_\pi) = c_{\nu\pi}^{(+s)}$  is the amplitude of probability to find a certain  $1^-$  neutron-(proton-hole) configuration in the collective state  $|s^{(+)}, 1M\rangle$ .

The pole representation of the effective field, strength function, and also nucleon-escape amplitudes is valid in the energy region of an isolated doorway-state resonance corresponding to the respective (quasi-stationary) collective

state. As applied to the IAR with the excitation energy higher the proton separation threshold, the Breit-Wigner-type parametrization of the Fermi strength function  $S_F(\omega^{(-)})$  and partial proton-escape amplitudes  $M_{F,\nu}(\omega^{(-)})$  are explicitly given by Eq. (9) of Ref. [6]. (See also Eq. (6) of Ref. [5]). We show here only the representation of the Fermi effective field:

$$\tilde{V}_F(r, \omega^{(-)}) \rightarrow \frac{2F'}{r^2} \frac{S_A^{1/2} \rho_A(r)}{\omega^{(-)} - \omega_A^{(-)} + i\frac{\Gamma}{2}}. \quad (6)$$

Here  $\rho_A(r) = \sqrt{4\pi} r^2 \rho_A(\vec{r})$  is the IAS transition density;  $S_A^{1/2} = \sqrt{4\pi} \int \rho_A(r) dr$ ,  $S_A$  is the IAS Fermi strength;  $\Gamma = \sum_\nu \Gamma_\nu$  is the IAR total proton width, while  $\Gamma_\nu$  is the partial width for IAR proton decay into the neutron-hole state  $\nu^{-1}$  of the product nucleus. Note that the “ideal”-IAS transition density  $\rho_A^{id}(\vec{r}) = (N - Z)^{-1/2} n^{(-)}(\vec{r})$  is proportional to the neutron excess density. ( $S_A^{id} = (N - Z)$ , the neutron excess is related to the parent nucleus).

### C. $(\gamma p)$ -reaction amplitude

The expression for the amplitude of the  $(\gamma p)$ -reaction, accompanied by DIAR excitation, is derived in three steps. First, we derive the CRPA expression for the amplitude of the IAR partial proton width,  $(\Gamma_\nu)^{1/2}$ . This expression can be obtained from Eqs. (8), (9) of Ref. [6] (or from Eqs. (5), (6) of Ref. [5]) and the pole representation of the Fermi effective field of Eq. (6):

$$(\Gamma_\nu)^{1/2} = \sqrt{2\pi} N_\nu^{1/2} \delta_{(\pi)(\nu)} \int \chi_{\varepsilon(\pi)}(r) v_A^{tr}(\vec{r}) \chi_\nu(r) dr. \quad (7)$$

Here,  $N_\nu = (2j_\nu + 1)n_\nu$  is the number of neutrons filling level  $\nu$  in a parent-nucleus state;  $r^{-1} \chi_{\varepsilon(\pi)}(r)$  is the normalized to the  $\delta$ -function of energy radial continuum-state (real) wave function for the escaping proton with the energy  $\varepsilon = \varepsilon_\nu + \omega_A^{(-)}$ ;  $v_A^{tr}(\vec{r}) = 2F' \rho_A(\vec{r})$  is the IAS transition potential. For the “ideal” IAS  $\omega_A^{(-)} = \Delta_C$ , and the transition potential  $(v_A^{tr})^{id} = (N - Z)^{1/2} v$  is proportional to the mean-field isovector part, provided that the isospin-selfconsistency condition is fulfilled:  $v(r) = 2F' n^{(-)}(\vec{r})$ , where  $v(r)$  is the symmetry potential (see, e.g., Ref. [6]).

The above-outlined CRPA expression of Eq. (7) can be applied to the IAS based on the “valence neutron + closed-shell core” parent-nucleus state. (In such a case  $N_\nu = 1$ ). This point allows us to generalize Eq. (7) for the case of the collective neutron-(proton-hole)-type parent states  $|s^{(+)}, 1M\rangle$ . Considering the IAS as the “ideal” one, we can substitute the product  $N_\nu^{1/2} \chi_\nu(r)$  in Eq. (7) by the sum  $\sum_\nu c_{\nu\pi}^{(+s)} \chi_\nu(r) n_\pi^{(1/2)}$  to get the expression for the doubly-partial amplitude of the DIAR proton width:

$$(\Gamma_{(\pi')\pi}^s)^{1/2} = \sqrt{2\pi} (2T_>)^{-1/2} n_\pi^{1/2} t_{(\pi')(\pi)}^{(1)} \int \chi_{\varepsilon(\pi')}(r) v(r) g_{(\pi')}(r, r', \varepsilon_\pi + \omega_s^{(+)}) 2F' \rho_s^{(+)}(r') \chi_\pi(r') \frac{dr dr'}{r'^2}. \quad (8)$$

Here,  $\varepsilon = \varepsilon_\pi + \omega_s^{(+)} + \Delta_C$  is the energy of the escaping proton, the symmetry potential  $v(r)$  is defined for the  $(Z - 1, N + 1)$  parent nucleus,  $(rr')^{-1} g_{(\pi')}(r, r', \varepsilon')$  is the radial neutron Green function with  $(\nu) = (\pi')$ . In derivation of Eq. (8) we use the relation between the probability amplitude  $c_{\nu\pi}^{(+s)}$  and corresponding transition-density element of Eq. (5) and, also, the spectral expansion for the radial neutron Green function. The partial width for DIAR proton decay into the proton-hole state  $(\pi)^{-1}$  of the product nucleus  $(Z - 1, N)$  equals  $\Gamma_\pi^s = \sum_{(\pi')} \Gamma_{(\pi')\pi}^s$ . Here, the sum is taken over the proton continuum-state quantum numbers  $(\pi') = j_{\pi'}, l_{\pi'}$ , which are compatible with the selection rules for the dipole transitions. It should be noted, that the use of Eq. (7) for derivation of Eq. (8) means, in fact, the use a perturbation theory on DIAR coupling to the single-proton continuum.

Using the expression of Eq. (8) for the doubly-partial DIAR proton-decay amplitude, one can obtain the CRPA-based expression for the corresponding doubly-partial  $(\gamma p)$ -reaction amplitude  $R_{(\pi')\pi}(\omega)$ . For this purpose, we use the pole representation of Eq. (4) for the dipole effective field and dipole strength function and also the relation of Eq. (2) between the  $\text{GDR}_>$  and  $\text{GDR}^+$  strength functions. The result is:

$$R_{(\pi')\pi}(\omega) = (B\omega)^{1/2} (2T_>)^{-1} n_\pi^{1/2} t_{(\pi')(\pi)}^{(1)} \int \chi_{\varepsilon(\pi')}(r) v(r) g_{(\pi')}(r, r', \varepsilon_\pi + \omega_C^{(+)}) \tilde{V}(r', \omega_C^{(+)}) \chi_\pi(r') dr dr', \quad (9)$$

where  $\varepsilon = \varepsilon_\pi + \omega$ . The pole representation of this amplitude is consistent with Eq. (8):

$$R_{(\pi')\pi}(\omega) \rightarrow \sqrt{\frac{3\pi^2}{2}} \frac{\hbar c}{\omega_s} \frac{(\Gamma_{\gamma_0,s})^{1/2} (\Gamma_{(\pi')(\pi)}^s)^{1/2}}{\omega - \omega_s + i0}. \quad (10)$$

Being based on Eqs. (9), (10), and the expression for the DIAR elastic radiative width (Subsect. II A), we can evaluate within the CRPA the partial proton widths for a given DIAR. An expression similar to Eq. (9) has been derived in Ref. [8] in an intuitive way.

### III. PROPERTIES OF THE DIAR IN $^{90}\text{Zr}$

#### A. Input quantities and model parameters

A nuclear mean field and particle-hole interaction are the input quantities for any CRPA-based approach. Along with the phenomenological (non-spin-flip and momentum-independent) Landau-Migdal particle-hole interaction, we use a realistic phenomenological isoscalar part of the nuclear mean field (including the spin-orbit term), while the mean-field isovector part (the symmetry potential) and also the mean Coulomb field are calculated selfconsistently (see, e.g. Refs. [6, 9]). The parametrization of the mean-field isoscalar part is explicitly given in Ref. [9]. To describe better the neutron single-quasi-particle spectrum in  $^{89,91}\text{Zr}$ , the strength parameters are somewhat specified:

$$U_0 = 54.9\text{MeV}, \quad U_{S0} = 13.85\text{MeV fm}^2, \quad f' = 1.03, \quad (11)$$

where  $F' = f' 300 \text{ MeV fm}^3$ .

Following Ref. [3], we take effectively into account the contribution of the isovector momentum-dependent forces in formation of the  $\text{GDR}^{(\pm)}$  by the scaling transformation of the CRPA strength functions:  $S^{(\pm)}(\omega^{(\pm)}) \rightarrow (1 + \tilde{k}')^{-1} S^{(\pm)}(\omega^{(\pm)}) / (1 + \tilde{k}')$ . The parameter  $\tilde{k}'$  describes the relative contribution of the momentum-dependent forces to the corresponding energy-weighted sum rule. For  $^{90}\text{Zr}$  the  $\tilde{k}' = 0.1$  value has been found in Ref. [3] from independent data [10]. In the properly modified CRPA calculations of the photo-absorption cross section  $\sigma_a^>(\omega)$  (see also Subsect. II A), performed in Ref. [3] for  $^{48}\text{Ca}$  and  $^{90}\text{Zr}$ , a few isolated DIAR have been found.

To improve description of the DIAR in  $^{90}\text{Zr}$ , in the present work we take proton pairing approximately into account by simply substituting in CRPA equations the occupation numbers  $n_\pi$  by the Bogolyubov coefficients  $v_\pi^2$ . The same modification is used to calculate the neutron-excess density in order to keep the isospin-selfconsistency condition [6, 11]. The energy gap parameter is taken 1.0 MeV (this value is close to that used in Ref. [11]), while the proton chemical potential is properly calculated to find, finally, the coefficients  $v_\pi^2$ .

#### B. Calculation results

We calculate first the photo-absorption cross section  $\sigma_a^>(\omega)$  for the excitation energy interval 14–18 MeV, which corresponds to the low-energy part of the  $\text{GDR}^{(+)}$  strength function of Eq. (2) [3]. In strength function calculations (performed with and without taking proton pairing into account) we add an artificial small imaginary part to the single-particle potential, used within the CRPA for calculations of the  $\omega$ -dependent quantities, to make presentation of the results more convenient (Fig. 1). Within the mentioned interval we find a few DIAR, whose energies  $\omega_s$  and calculated via  $(\sigma_a^>)_s^{int}$  radiative widths  $\Gamma_{\gamma_0,s}$  are given in Table I. The particle-hole part of the DIAR transition density  $\rho_{>,s}(r) = (2T_{>})^{(-1/2)} \rho_s^{(+)}(r)$  is next calculated via the residues at the poles in the energy dependence of the dipole effective field of Eq. (4). The calculated transition densities are shown in Fig. 2. These densities are used to evaluate within the improved plane wave approximation the DIAR form factor of Eq. (3). The results are shown in Fig. 3. Further, Eqs. (9), (10) are used to evaluate the DIAR partial proton widths  $\Gamma_{\pi_0-\pi_3}$  for population of the  $2p_{1/2}$  ( $\pi_0$ ),  $1g_{9/2}$  ( $\pi_1$ ),  $2p_{3/2}$  ( $\pi_2$ ),  $1f_{5/2}$  ( $\pi_3$ ) one-hole states in  $^{89}\text{Y}$ . The calculated widths are given in Table I together with the experimental data of Ref. [4], related to the certain DIAR ( $E_x = 16.1 \text{ MeV}$ ). For this DIAR we show also the calculated doubly-partial proton widths  $\Gamma_{(\pi')\pi}^s$  for decay into the main channels ( $\pi_0$ ) and ( $\pi_2$ ) (Table II).

Using the calculated DIAR total proton width  $\Gamma_s = \sum_\pi \Gamma_\pi^s$  we can evaluate the  $(\gamma p_{tot})$ -reaction cross section for the considered excitation-energy interval:

$$\sigma_{\gamma p_{tot}}^>(\omega) = \frac{1}{2\pi} \sum_s \frac{(\sigma_a^>)_s^{int} \Gamma_s}{(\omega - \omega_s)^2 + \frac{1}{4} \Gamma_s^2}. \quad (12)$$

To compare with the experimental data of Ref. [11], we make “apparatus” averaging this cross section with the function  $I(\omega, \omega') = (I/\pi) / ((\omega - \omega')^2 + I^2)$ . The averaged cross sections, obtained for the appropriate  $I$  values ( $I = 50$  and  $100 \text{ keV}$ ), are shown in Fig. 4. together with the experimental data of Ref. [12]. In calculations of the mentioned cross section the relatively small contribution of the  $\text{GDR}_<$  excitation is also taken into account. It is done within the similar approach by the methods described in Ref. [10].

### C. Discussion of the results

Due to the specific shell-model structure of  $^{90}\text{Zr}$ , the proton pairing in this nucleus is rather weak. In particular, the number of protons, occupying the  $\pi$  level,  $N_\pi = (2j_\pi + 1) v_\pi^2$ , is relatively small for the  $1g_{9/2}$  level. (In Table II the  $N_\pi$  values are shown for a few levels near the proton chemical potential). Nevertheless, due to pairing, the DIAR with the energy 16.3 MeV appears in calculations (Fig. 1) and DIAR proton decay into the  $9/2^+$ -state of  $^{89}\text{Y}$  becomes formally possible (Table I). Among the IAR found in calculations, three of them have relatively large elastic radiative width (Table I), transition density (Fig. 2) and, therefore, the form factor (Fig. 3). The DIAR with the energy 16.1 MeV can be related to the resonance experimentally studied in Ref. [4]. Within the present approach we obtained a satisfactory description of the decay properties of this DIAR (Table I). The  $d_{3/2}$  component of the proton partial widths is found to be the main one for decay into the  $\pi_0$ -decay channel. This observation is in agreement with the analysis of the DIAR wave function performed in Ref. [4]. However, it is not truth for another main decay channel ( $\pi_2$ ) as it follows from Tables I and II. The two-node transition densities of all DIAR (Fig. 2) show that the low-energy  $1^-$  parent states are not strongly collectivized, as it expected for the pygmy resonance. (The main-peak  $\text{GDR}^{(+)}$  transition density is nodeless). The form factor, calculated for the DIAR with the energy 16.1 MeV, is in reasonable agreement with experimental data of Ref. [4]. The calculation results show that at least two DIAR (with the calculated energy 16.7 and 17.2 MeV) can be found experimentally via the  $(e, e'p)$ -reaction. This statement is partially supported by comparing the calculated and experimental  $(\gamma p_{\text{tot}})$ -reaction cross section within the excitation-energy interval 14–18 MeV (Fig. 4). It is noteworthy that the approximations used in description of the IAS, proton pairing, and single-particle level scheme lead to uncertainties in evaluation of the DIAR energies (presumably, in a few hundreds keV). Description of the DIAR decay properties seems to be not much sensitive to these uncertainties.

In conclusion, we first present continuum-RPA-based description of the decay properties of the specific doubly-collective states,  $1^-$ -IAR, and describe satisfactorily the available experimental data.

### Acknowledgement

The authors would like to thank M.N. Harakeh and P. von Neumann-Cosel for interesting discussions and valuable remarks. One of the authors (M.H.U.) thanks M.N. Harakeh for warm hospitality during a long-term stay at KVI and acknowledges the support from the “Nederlandse organisatie voor wetenschappelijk onderzoek” (NWO). This work is partially supported by the Russian Fund for Basic Researches (RFBR) under the grant No. 06-02-016883-a.

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TABLE I: The energies (in MeV), elastic radiative and partial protons widths (in eV and keV, respectively), calculated for the isolated DIAR in  $^{90}\text{Zr}$ . The parameters deduced in Ref. [4] for the observed DIAR (the widths are given with errors) are also shown.

$\omega_s$	$\Gamma_{\gamma_0}$	$\Gamma_{\pi_0}$	$\Gamma_{\pi_1}$	$\Gamma_{\pi_2}$	$\Gamma_{\pi_3}$
14.91	68	49	0.1	8	—
16.09	110	55.1	1.7	10.6	0.12
16.28	$108 \pm 35$	$50 \pm 6$	$< 2.0$	$20 \pm 5$	$7 \pm 2$
16.35	66	8	6.5	20	0.10
16.73	169	6	0.4	159	0.02
17.19	185	8	0.3	81	0.06

TABLE II: The doubly-partial widths (in keV), calculated for the main channels of proton decay of the DIAR at 16.1 MeV in  $^{90}\text{Zr}$ .

$\pi$	$2p_{1/2}$			$1g_{9/2}$			$2p_{3/2}$			$1f_{5/2}$		
$N_\pi$	1.49			0.83			3.77			5.89		
$(\pi')$	$s_{1/2}$	$d_{3/2}$	$f_{7/2}$	$h_{9/2}$	$h_{11/2}$	$s_{1/2}$	$d_{3/2}$	$d_{5/2}$	$d_{3/2}$	$d_{5/2}$	$h_{7/2}$	
$\Gamma_{(\pi'),\pi}$	0.0	55.1	0.1	0.0	1.6	2.2	3.8	4.6	0.01	0.10	0.0	

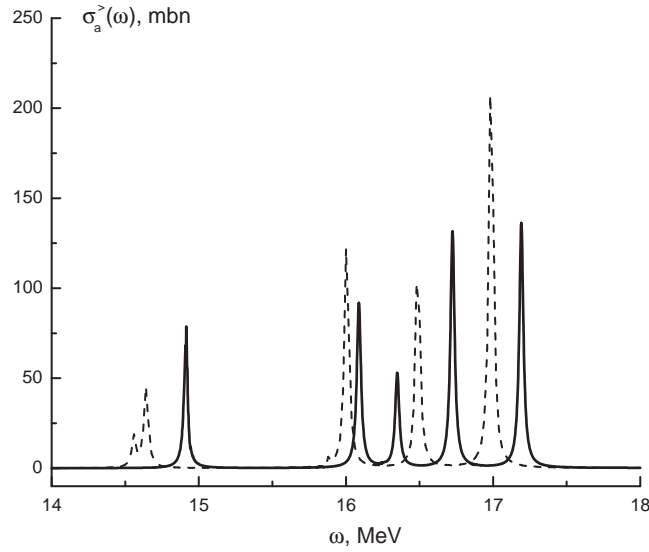


FIG. 1: The cross sections of photo-absorption, accompanied by excitation of the isolated DIAR in  $^{90}\text{Zr}$ . The full and dotted lines correspond, respectively, to calculations with and without taking the proton pairing into account.

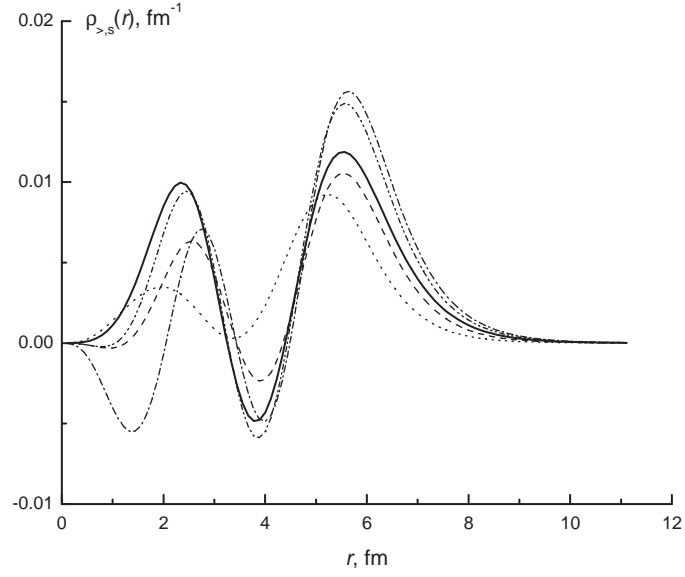


FIG. 2: The calculated proton-(proton-hole) part of the DIAR transition density. The dashed, full, pointed, dashed-dotted and dashed-dot-dotted lines correspond to the DIAR at 14.9, 16.1, 16.3, 16.7, 17.2 MeV, respectively.

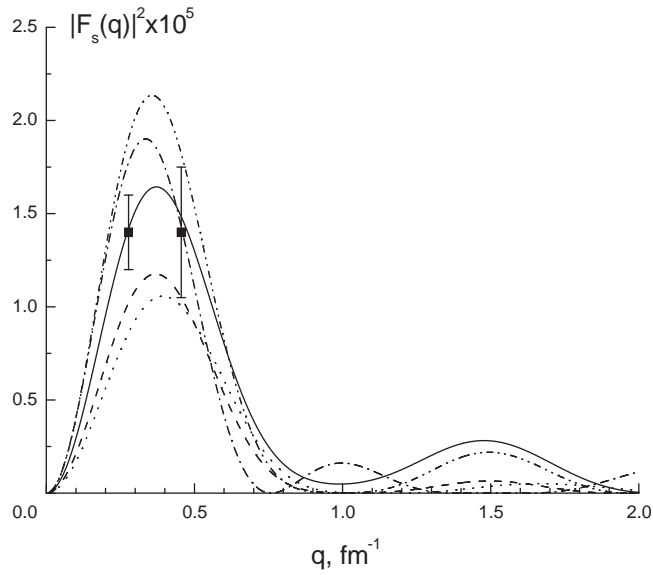


FIG. 3: The DIAR form factors, calculated within the improved plane wave approximation. The notations are the same, as in Fig. 2. The experimental data concerned with the DIAR at 16.1 MeV are taken from Ref. [4].



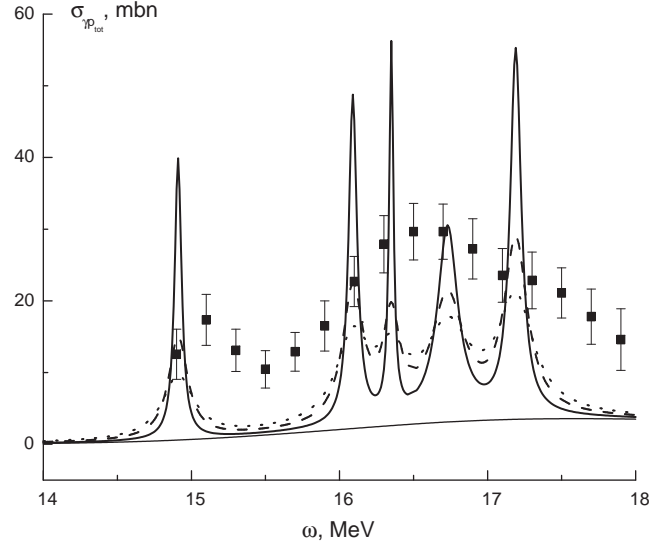


FIG. 4: The  $^{90}\text{Zr}(\gamma p_{tot})$ -reaction cross section, calculated for the excitation-energy interval 14–18 MeV (full line). The dashed and dotted correspond to averaging with the parameter  $I$  equal 50 and 100 keV, respectively. The thin line is for  $\text{GDR}_{<}$  contribution to the cross section. The experimental data are taken from Ref. [12].